Generation of one-cycle laser pulses by use of high-amplitude plasma waves

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The dynamics of a short laser pulse located in the density trough of a background plasma wave is investigated and a scheme is proposed to compress the pulse duration by use of a high-amplitude plasma wave. The threshold amplitude of the plasma wave, at which the compressing effect just balances the dispersive spreading of the laser pulse, is estimated for certain pulse profiles. Numerical simulations are conducted with particle-in-cell codes, where a pump pulse is used to generate a high-amplitude plasma wave and a signal pulse copropagates behind. It is shown that the signal pulse can be compressed by the plasma wave from ten laser cycles to about one cycle within a millimeter in tenuous plasma only a few percent of the critical density.

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I. INTRODUCTION

The generation of ultrashort intense laser pulses is crucial for various applications, such as the fast ignition scheme for the inertial confined fusion [1], particle acceleration [3,4], laser-induced nuclear reactions [2], ultrashort x-ray sources [5], etc. To the advantage of these applications, the short-pulse laser technology has progressed very rapidly in the last decade [6]. The pulse duration has decreased steadily, now down to several femtoseconds [7], and the focused intensity has increased up to 10^{21} W/cm^2 [8]. As is well known, these progresses are usually realized making use of conventional optical materials and components. Recently it has been proposed to compress short pulses, making use of the plasma nonlinearity of ionizing gas [9], and to amplify and compress short pulses in plasmas through stimulated Raman scattering [10,11]. In the latter scheme, a pump pulse is injected in the counterpropagating direction of a signal pulse, and the modulated electron density serves as the intermediator for energy transfer from the pump to the signal pulse. In comparison with the chirped-pulse amplification (CPA) technology, it allows for simultaneous pulse compression and energy amplification.

In this paper we propose an alternative way of compressing a short laser pulse about a duration of $2\pi/\omega_p$ in a plasma, where $\omega_p=(4\pi n_0 e^2/m)^{1/2}$ is the plasma frequency. In this scheme, we use a pump pulse to produce a background plasma wave while a signal pulse copropagates behind the pump. It is well known that the moving density gradients of the plasma wave can serve as a phase modulator producing a frequency upshift or downshift of a short laser pulse [12–15]. We tune the time interval between the signal pulse and the pump so that the signal pulse is located at the bottom of a density trough of the background plasma wave. In this case, the leading edge of the signal pulse is located in a region with an increasing density profile and its local frequency is downshifted. The trailing edge is located in a region with a decreasing density profile and its local frequency is upshifted [13]. Therefore the local group velocity of the pulse decreases in its leading edge and increases in its trailing edge. As a result, the pulse can be compressed when the amplitude of the plasma wave is high enough so that the compressing effect can overcome the inherent dispersive spreading of the pulse [16]. Near some threshold amplitude, the signal pulse neither spreads nor is compressed, forming a solitarylike wave. Recalling earlier theories on solitary waves in plasmas [17], one finds that solitary waves are accompanied by a density depression, which modulates the wave frequency and is responsible for the formation of solitary waves. Such a kind of solitary waves, which even though have been observed in particle-in-cell simulations for certain parameter regimes [18], cannot be controlled in numerical simulations and experiments. In the present case, the density depression is provided externally by the plasma wave, which can be controlled by tuning the relative phase between the signal pulse and the plasma wave. Also one can control the pulse width by changing the amplitude of the plasma wave and plasma densities. The present scheme also allows for compressing laser pulses at relativistically high intensities without the stretching step to avoid damaging optical components in CPA technology. The outline of the paper is as follows. In Sec. II, the scheme is analyzed and the threshold amplitude of the plasma wave for pulse compression is estimated. In Sec. III, this scheme is studied numerically with particle-in-cell (PIC) codes. Pulse compression both in homogeneous and inhomogeneous plasmas is investigated and the possibility of compressing a high-frequency laser pulse in an inhomogeneous plasma is examined. The paper concludes with a brief discussion in Sec. IV.

II. THRESHOLD AMPLITUDE OF PLASMA WAVES FOR PULSE COMPRESSION

For a laser pulse propagating in homogeneously tenuous plasma, the one-dimensional envelope equation can be written as [16,19]

$$\left(2i\frac{\partial}{\partial\tau}+2i\,\delta v\,\frac{\partial}{\partial\xi}+2\beta\epsilon\frac{\partial^2}{\partial\xi\partial\tau}+\epsilon^2\frac{\partial^2}{\partial\xi^2}\right)a=\left(\frac{n}{\gamma}-1\right)a, \tag{1}$$

where a is the slowly varying amplitude of the laser pulse, n the electron density normalized by the unperturbed frequency n_0, γ the relativistic factor, $\beta=v_p/c$ the phase velocity of a background plasma wave, $\delta v=(\omega_0/\omega_p)(v_g-v_p)/c\ll\beta$ with v_g the group velocity of the signal pulse, $\epsilon=\omega_p/\omega_0\equiv(n_0/n_c)^{1/2}$ with $\omega_p=(4\pi n_0 e^2/m)^{1/2}$ the unperturbed plasma frequency and ω_0 the laser frequency, $\tau=t\omega_p^2/\omega_0$, and $\xi=k_p(x-\beta ct)$ with $k_p=\omega_p/c$. In the quasistatic approximation [20], the electron density in the plasma wave can be written as $n=\beta^2\gamma/(1+\phi-\gamma\gamma_\beta^{-2})$ with $\gamma=\gamma_\beta^2[1+\phi-\beta\sqrt{(1+\phi)^2-\gamma_\beta^{-2}(1+|a|^2/2)}]$, $\gamma_\beta=(1-\beta^2)^{-1/2}$, and the scalar potential ϕ determined by [20,21]

$$\frac{\partial^2\phi}{\partial\xi^2}=-\frac{1+\phi-\gamma}{1+\phi-\gamma_\beta^{-2}\gamma}. \tag{2}$$

In the weakly relativistic approximation for $|a|\ll1$ and $|\partial\phi/\partial\xi|\ll1$, we have $n/\gamma=1+\mathcal{L}\phi$ with $\mathcal{L}=\gamma_\beta^{-2}\partial^2/\partial\xi^2-1$ and ϕ determined by $(\beta^2\partial^2/\partial\xi^2+1)\phi=|a|^2/4$, giving

$$\phi=\phi_b(\xi)+\frac{1}{4\beta}\int_{\infty}^{\xi}|a(\tau,\xi'-\xi_0)|^2\sin\left(\frac{\xi-\xi'}{\beta}\right)d\xi'. \tag{3}$$

Here $\phi_b=-\beta^2\delta n_0\cos(\xi/\beta)$ is a backgroud plasma wave, which could be generated by another laser pulse, for example. The second term is the wake field ϕ_w generated by the laser pulse itself assuming that the pulse is centered at $\xi=\xi_0$.

Defining $a=|a|\exp[i\theta(\xi,\tau)]$ and the phase $\psi=k_0x-\omega_0 t+\theta$, then the local frequency of the pulse is given by $\omega=-\partial\psi/\partial t\approx\omega_0(1+\beta\epsilon\partial\theta/\partial\xi)\equiv\omega_0+\delta\omega$. One obtains from Eq. (1) that

$$\frac{dW}{d\tau}=\frac{\beta\epsilon}{2}\int_{-\infty}^{+\infty}\left[|a|^2\frac{\partial}{\partial\xi}\left(\frac{n}{\gamma}\right)+2\beta\epsilon\frac{\partial}{\partial\tau}\left|\frac{\partial a}{\partial\xi}\right|^2\right]d\xi$$
$$\equiv-\beta\epsilon\frac{d}{d\tau}\int_{-\infty}^{+\infty}|a|^2\frac{\partial\theta}{\partial\xi}d\xi, \tag{4}$$

where $W=\int|a|^2d\xi$. Equation (4) leads to $\int\omega|a|^2d\xi=$const, which means that the photon number $\sim\int(|E|^2/\omega)d\xi$ is conserved [19]. We emphasize that this is valid independent of the weakly relativistic approximation. Next we seek conditions at which the central frequency and the pulse energy do not change by setting $dW/d\tau=0$, which leads to

$$\delta n_0\sin(\xi_0/\beta)=\frac{1}{8\beta^3}\int_{-\infty}^{+\infty}|a(\tau,\xi')|^2\cos\left(\frac{\xi'}{\beta}\right)d\xi'\equiv I_1 \tag{5}$$

if the pulse remains symmetric about $\xi=\xi_0$. As a result, one obtains $\xi_0=\beta\pi-\beta\sin^{-1}(I_1/\delta n_0)$ or $\xi_0=\beta\sin^{-1}(I_1/\delta n_0)$. It means that if the laser pulse is centered near a density trough or peak of the *total* plasma wave, a superposition of the background plasma wave ϕ_b and the wake field of the pulse ϕ_w, there is not net energy exchange between the pulse and the background plasma wave. In this case, the energy that the laser pulse obtains from the background plasma wave equals that it loses to the exciting the wake field. Symmetric pulse compression occurs near the density trough, while symmetric pulse spreading is found near the density peak. Alternatively, if the laser pulse is centered at a density trough of the *total* plasma wave, one should have $\partial(n/\gamma)/\partial\xi=0|_{\xi=\xi_0}$, which leads to Eq. (5) as well as the pulse being symmetric. Obviously, if one neglects the wake field excitation, when $I_1\ll\delta n_0$, Eq. (5) simply gives $\xi_0=0$ or $\beta\pi$ [16].

Now we consider the dynamics of the pulse when it is located near a density trough of the background plasma wave. Defining $\langle\xi\rangle=W^{-1}\int\xi|a|^2d\xi$, $\langle\xi^2\rangle=W^{-1}\int\xi^2|a|^2d\xi$, and $\langle\delta\xi^2\rangle=\langle\xi^2\rangle-\langle\xi\rangle^2$, and assuming $dW/d\tau=0$, one finds

$$\frac{d\langle\xi\rangle}{d\tau}=\delta v+\frac{i\epsilon^2}{W}\int\left(a\frac{\partial a^*}{\partial\xi}\right)d\xi\equiv\delta v+\frac{\epsilon^2}{W}\int|a|^2\frac{\partial\theta}{\partial\xi}d\xi, \tag{6}$$

$$\frac{d^2\langle\xi\rangle}{d\tau^2}=-\frac{\epsilon^2}{2W}\int\left(|a|^2\frac{\partial}{\partial\xi}\mathcal{L}\phi+2\beta\epsilon\frac{\partial}{\partial\tau}\left|\frac{\partial a}{\partial\xi}\right|^2\right)d\xi, \tag{7}$$

$$\frac{d^2\langle\delta\xi^2\rangle}{d\tau^2}=\frac{\epsilon^2}{W}\int\left[2\epsilon^2\left|\frac{\partial a}{\partial\xi}\right|^2-|a|^2\xi\frac{\partial}{\partial\xi}(\mathcal{L}\phi)\right]d\xi-\frac{d^2\langle\xi\rangle^2}{d\tau^2}$$
$$+2\delta v\left(2\frac{d\langle\xi\rangle}{d\tau}-\delta v\right), \tag{8}$$

where we assume $d\delta v/d\tau=0$. Equations (6) and (7) describe the motion of the pulse center and the frequency shift of the pulse. From Eq. (7) one obtains

$$\frac{d^2\xi_0}{d\tau^2}=\frac{4\epsilon^2\beta^2\delta n_0 I_1}{W}\sin(\xi_0/\beta). \tag{9}$$

If $\delta v=0$, Eq. (9) describes the oscillating motion of the pulse packet at the density trough around $\xi_0=\beta\pi$ with a bounce frequency $\Omega\sim\delta n_0^{1/2}(\omega_p^3/\omega_0^2)$. The location of the pulse at the density peak $\xi_0=0$ is unstable to perturbations, leading either to acceleration (frequency up-shift) or to deacceleration (frequency downshift). Notice that these are very similar to the behaviors of a bunch of electron beams in the plasma wave.

Equation (8) describes the dynamics of the pulse width in the background plasma wave, which is found to be independent of δv when substituting $d^2\langle\xi\rangle^2/d\tau^2$ with Eqs. (6) and (7). If the pulse profile is $|a|^2=a_0^2\cos^2[\pi(\xi-\xi_0)/L]$ with $|\xi-\xi_0|\le L/2$ and L the spatial width of the pulse normalized by k_p^{-1}, Eq. (8) gives

$$\frac{d^2\langle\delta\xi^2\rangle}{d\tau^2}=\frac{2\epsilon^2}{L}$$
$$\times\left[\frac{\epsilon^2\pi^2}{L}+F_0(L,\beta)\delta n_0\cos\left(\frac{\xi_0}{\beta}\right)+a_0^2F_1(L,\beta)\right], \tag{10}$$

where $F_0=\beta\eta_1^2\eta_3\sin(L/2\beta)-(\eta_1 L/2)\cos(L/2\beta)>0$ for $L<2\beta\pi$, $F_1=(\eta_1/16\beta^2)[2F_0\cos(L/2\beta)+3L^3/16\pi^2\beta^2+3\gamma_\beta^{-2}L/4\eta_1]$, $\eta_1=[1-(L/2\pi\beta)^2]^{-1}$, and $\eta_3=1-3(L/2\pi\beta)^2$. The threshold amplitude at which pulse spreading is balanced by the compression effect of the plasma wave is found by setting $d^2\langle\delta\xi^2\rangle/d\tau^2\le0$, i.e.,

$$\delta n_0\ge[LF_0|\cos(\xi_0/\beta)|]^{-1}(\pi^2\epsilon^2+a_0^2LF_1). \tag{11}$$

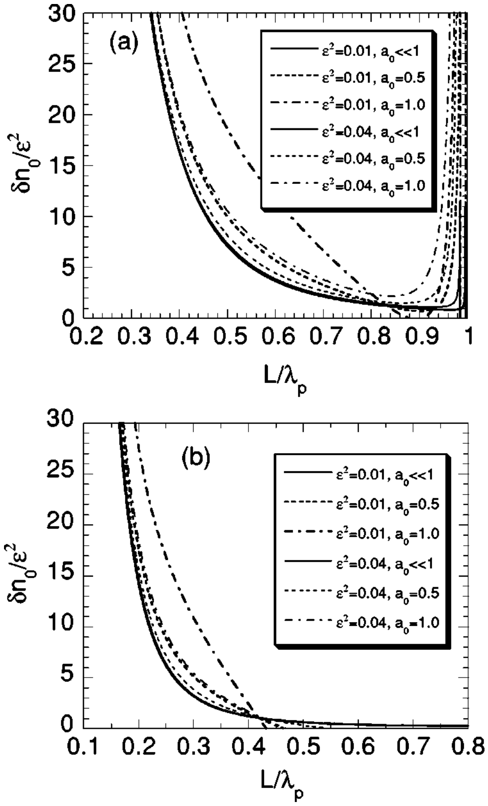

FIG. 1. Threshold amplitudes of plasma waves (δn_0 normalized by the unperturbed plasma density) as a function of pulse widths for pulses with sine (a) and Gaussian (b) profiles at different peak intensities.

Together with the optimized position from Eq. (5) with $I_1 = (\eta_1 a_0^2/8\beta^2)\sin(L/2\beta)]$, Eq. (11) gives the threshold amplitude of the background plasma wave for the given pulse profile, which have taken into account both the dispersive spreading effect (proportional to $\pi^2\epsilon^2$) and the effect of the self-generated wake field (proportional to $a_0^2 L F_1$). Figure 1(a) shows the threshold as a function of the pulse width. Obviously it increases with decreasing pulse width. It is relatively low for a pulse with width around $L = 2\pi\beta$ (or dimensionally $L = \lambda_p$ with $\lambda_p = 2\pi\beta/k_p$). Usually the wake field term tends to increase the threshold as the pulse intensity increases. It suggests that one needs a high-amplitude plasma wave to compress a short laser pulse at high intensity to a very narrow width less than $0.3\lambda_p$. From Eq. (10) one finds that the time scale of pulse compression is about $\sim (L^2/\pi)\omega_0^3/\omega_p^4$. For other pulse profiles such as the Gaussian pulse $|a|^2 = a_0^2 \exp[-4(\xi-\xi_0)^2/L^2]$, one obtains in a similar way from Eq. (8) that

$$\frac{d^2\langle \delta\xi^2\rangle}{d\tau^2} = \frac{2\epsilon^2}{L}\left[\frac{2\epsilon^2}{L} + \frac{\delta n_0 L^3}{16\beta^2}\exp\left(-\frac{L^2}{16\beta^2}\right)\cos\left(\frac{\xi_0}{\beta}\right) + a_0^2 G_1(L,\beta)\right], \tag{12}$$

where

$$G_1 = (L^3/64\sqrt{2}\beta^4)(G_0 + 4\beta^2\gamma_\beta^{-2}/L^2)$$

and

$$G_0 = 1 - 2\sqrt{2/\pi}(\beta L)^{-1}\int_{-\infty}^{+\infty}dx\int_{+\infty}^{x}\exp[-4(x^2+x'^2)/L^2] \times \sin[(x-x')/\beta]dx'.$$

The threshold amplitude is given by

$$\delta n_0 \geqslant \frac{16\beta^2}{L^4}\exp\left(\frac{L^2}{16\beta^2}\right)\left|\cos\left(\frac{\xi_0}{\beta}\right)\right|^{-1}(2\epsilon^2 + a_0^2 L G_1). \tag{13}$$

Figure 1(b) plots the threshold amplitude as a function of the pulse width for Gaussian profiles. It increases very quickly as decreasing the pulse width for $L/\lambda_p < 0.5$. For any pulse profiles, it is obvious that a pulse must be precompressed to about one plasma wavelength or in time duration $\sim 2\pi/\omega_p$ before it is to be further compressed with this scheme. Pulses longer than one plasma wavelength will be split up into a pulse train.

III. NUMERICAL SIMULATIONS

The theory given in the last section is developed partially for a laser pulse at weakly relativistic light intensities and for a background plasma wave at weakly relativistic amplitudes, starting with the envelope equation. To confirm the theory, we have performed a series of numerical simulations with both one-dimensional (1D) and 2D PIC codes, which is free from these limitations on the theory, solving Maxwell's equations and the equation of motion for a few hundred thousand particles with the 1D code and about 10×10^6 particles with the 2D code [22]. In addition, PIC codes enable one to deal with pulse propagation and compression in inhomogeneous plasmas. In the simulations, we launch two laser pulses from the left boundary, where the first pulse is used as the pump to generate a background plasma wave and the second is a signal pulse following behind. One adjusts the time interval between them so that the second pulse is located inside a density trough of the wake field generated by the pump pulse.

A. Pulse compression in homogeneous plasmas

First we present the simulation results for pulse propagating in homogeneous plasmas, where the analytical theory given in last section applies. We start by checking the threshold amplitude at which the compression effect of the plasma wave can balance the dispersive spreading of the pulse. Both the pump and signal pulses are taken to be in sine profiles and in the same frequency. In Fig. 2, we take the plasma density $n_0/n_c = 0.01$. The initial pulse widths of the pump and signal are $10\lambda_0$ (or $10\tau_0$) and $5\lambda_0$ (or $5\tau_0$), respectively, where λ_0 is the laser wavelength in vacuum and τ_0 the laser cycle. The two pulses are separated by a time interval of about $34.8\tau_0$ initially, so that the signal pulse is located near a density trough center of the wake field generated by the pump pulse. We find that when the pump pulse takes a peak amplitude about $a_0 = 0.4$, the generated wake field can produce a necessary compression effect to balance the dispersive spreading, as shown in Figs. 2(a) and 2(b). Here the amplitude of the wake field is well given by the analytical

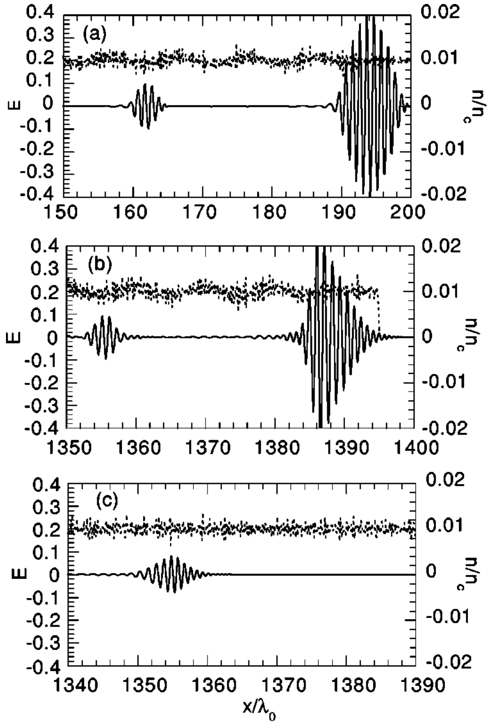

FIG. 2. Propagation of a signal pulse in a background plasma wave generated by a pump pulse in homogeneous plasma with density $n_0/n_c = 0.01$ at $t = 20\tau_0$ (a) and $1400\tau_0$ (b), where τ_0 is the laser cycle (the signal has the same frequency with the pump). The pump pulse is initially with $a_{01} = 0.4$ and $L_1 = 10\lambda_0$ and the signal pulse with $a_{02} = 0.1$ and $L_2 = 5\lambda_0$. Both the signal and pump pulses are in sine profiles with an initial interval of $34.8\tau_0$. (c) shows the signal pulse spreads without the pump pulse. The transverse field (E) is normalized by $m\omega_0 c/e$ with ω_0 the initial central frequency of the signal pulse.

result by $\delta n_0/n_0 = \eta_1 a_{01}^2 \sin(L_1/2\beta)/4 \approx \pi a_{01}^2/8 = 0.063$ for $L_1 = 2\pi\beta$ and $a_{01} = 0.4$ for the pump. It is also just close to the estimated threshold of about $2.25\pi\epsilon^2$ from Eq. (11). If there is not a pump propagating in front of the signal pulse, it will spreads as shown in Fig. 2(c).

To observe the compression effect, we increase the peak amplitude of the pump pulse up to $a_0 = 0.8$ so that a higher-amplitude plasma wave can be generated. As shown in Figs. 3(a)–3(c), the duration of the signal pulse reduces with the propagation distance. After propagating over $750\lambda_0$, the signal pulse initially with $10\tau_0$ is compressed to about only one laser cycle. This is also indicated by the corresponding frequency spectra, broadening with time as shown in Fig. 3(d). The one laser cycle is close to the compression limit since the laser pulse begins to spread afterwards if the plasma region extends longer in front of it. Notice that there are both frequency upshift and downshift in the frequency spectra, but the frequency downshift is larger than the upshift. As a result, the central frequency of the pulse is downshifted. This is caused by the forward motion of the pulse relative to the bottom of the density trough to the region with increasing density profile. Actually when the amplitude of the plasma wave is high, the plasma density at the trough is obviously lower than the unperturbed plasma density. Since the signal pulse and the pump are with the same frequency, the signal pulse can propagate with a higher velocity than the phase velocity of the background plasma wave, which is equal to the group velocity of the pump pulse in the unperturbed plasma. In the region with increasing density profile, the frequency of the laser pulse is downshifted [13]. For a narrow pulse, the frequency shift is approximately given by $\delta\omega/\omega_0 \sim \epsilon^3 \delta n_0 \sin(\xi_0/\beta)(\omega_0 t)/2 \sim 0.37$ when taking $\delta n_0 = 0.4$ and $\delta v = 0.2\epsilon$ and the pulse center changes $\xi_0 \sim \beta\pi + \delta v\,\tau$ in the first order according to Eq. (6). In the simulation shown in Fig. 3(d), the central frequency is downshifted by about 20% at $t = 500\tau_0$. Later, there is not much downshift in frequency when the pulse is strongly compressed and located in a region where the density gradient of the total plasma wave is small. During the compressing process, the pulse intensity is increased to more than double the initial one and a considerable part of its initial energy is preserved. Meanwhile, the central frequency is downshifted. Therefore $|E|^2 L/\omega$ is basically a conserved quantity, following from the envelope equation in Sec. II. Notice that the time scale for compression is about $\omega_0^3/2\omega_p^4$, consistent with the analytical estimation. If one increases the plasma density, the time scale

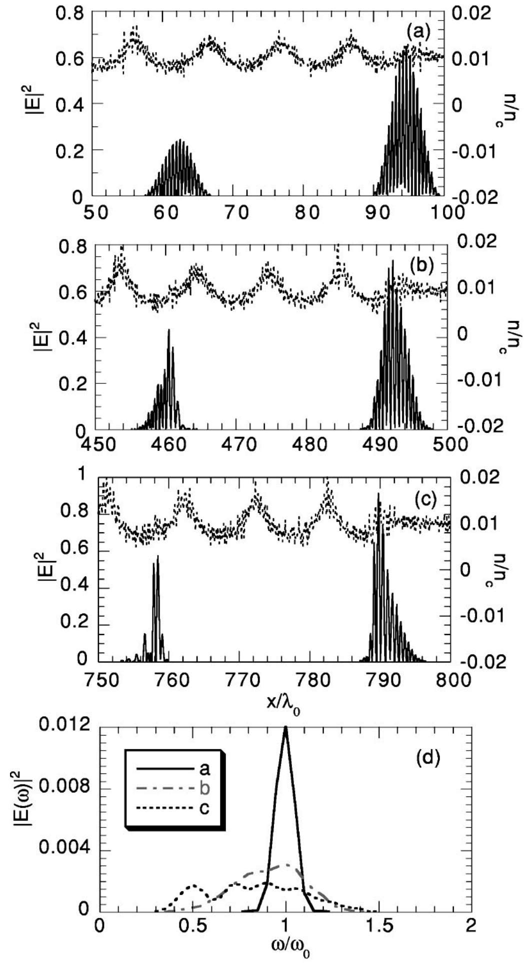

FIG. 3. Propagation of a signal pulse in a background plasma wave generated by a pump pulse in homogeneous plasma with density $n_0/n_c = 0.01$ at $t = 100\tau_0$ (a), $500\tau_0$ (b), and $800\tau_0$ (c). Initially $a_{01} = 0.8$ and $L_1 = 10\lambda_0$ for the pump pulse and $a_{02} = 0.5$ and $L_2 = 10\lambda_0$ for the signal pulse. Both the signal and pump pulses are in sine profiles with an initial interval of $32.5\tau_0$. The frequency spectra of the signal pulse at these snapshot moments are shown in (d). The transverse field (E) is normalized as in Fig. 2.

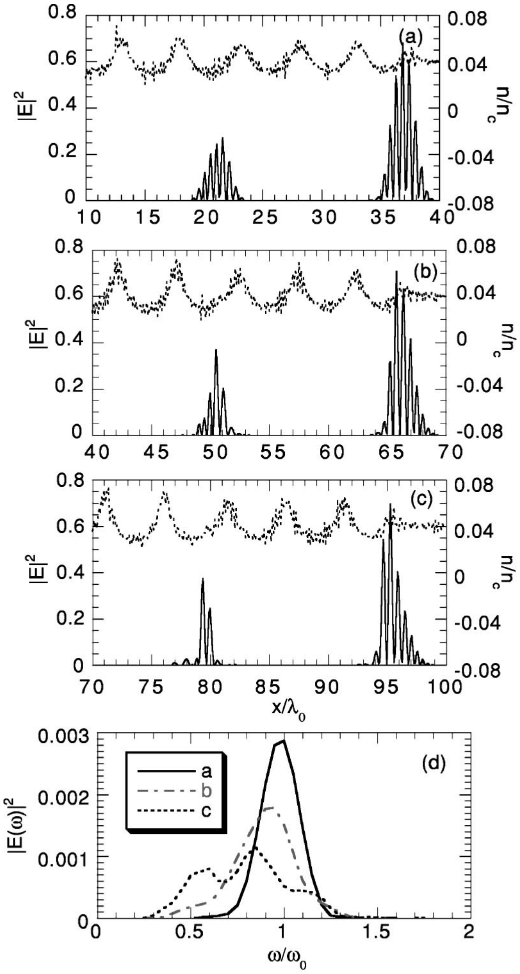

FIG. 4. Pulse compression in homogeneous plasma with density $n_0/n_c = 0.04$ at $t = 40\tau_0$ (a), $70\tau_0$ (b), and $100\tau_0$ (c). Initially $a_{01} = 0.8$ and $L_1 = 5\lambda_0$ for the pump pulse and $a_{02} = 0.5$ and $L_2 = 5\lambda_0$ for the signal pulse. Both the signal and pump pulses are in sine profiles with an initial interval of $16\tau_0$. The frequency spectra of the signal pulse at these snapshot moments are shown in (d). The transverse field (E) is normalized as in Fig. 2.

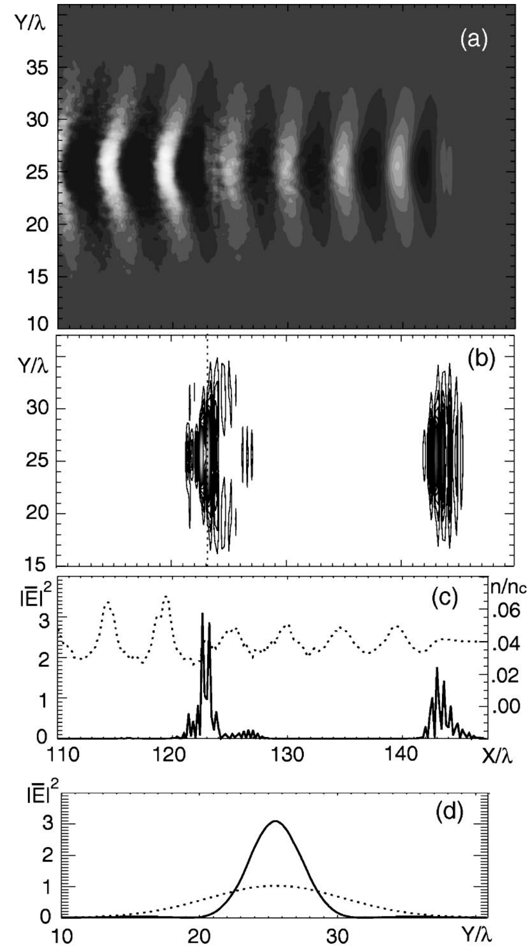

FIG. 5. Two-dimensional simulation results at $t = 150\tau_0$. Initially $a_{01} = 0.5$ and $L_1 = 5\lambda_0$ for the signal pulse, $a_{02} = 0.5$ and $L_2 = 5\lambda_0$ for the pump pulse, and $n_0/n_c = 0.04$. (a) The electron density scaling from 0.025 (black) to 0.065 (white) in unit of n_c, (b) the contour lines of the pulse energy density scaling from 0 to 3 in unit of the initial peak energy density, (c) the longitudinal profile of the electron density (dotted line, averaged in a laser cycle) and the electromagnetic energy density (solid line) at the center of the laser pulse ($Y = 25.5\lambda_0$), and (d) the transverse profile cut at about $x = 122\lambda_0$ [see the dashed line in (b)]. The initial transverse profile is shown by the dotted line. The transverse field (E) is normalized by the initial value or $0.5 m\omega_0 c/e$.

needed to compress the pulse is reduced. Such an example is shown in Fig. 4 where we take the plasma density $n_0/n_c = 0.04$. The pulse initially with five cycles is compressed to about one cycle after $80\tau_0$. In this example, the central frequency of the signal pulse is downshifted by about 15%.

We also have conducted simulations with a 2D PIC code [22] to see multidimensional effects. The plasma density is set to $n_0/n_c = 0.04$. Both the pump and signal pulses have a peak amplitude $a_{10} = a_{20} = 0.5$ and the initial pulse lengths are $L_1 = L_2 = 5\lambda_0$ (sine profile), respectively. Their transverse profiles are Gaussian with a full width at half maximum (FWHM) of about $10\lambda_0$. Figures 5(a) and 5(b) show contour plots of the electron density and the electromagnetic energy density, respectively, at $150\tau_0$. After propagating over about $150\lambda_0$, the signal pulse is strongly compressed down to about one laser cycle as shown in Figs. 5(b) and 5(c). Notice that the electron density contour lines are curved toward the axis with distance from the pump pulse. As is well known, this is due to the relativistic effect which tends to increase the plasma wavelength in the high-amplitude region close to the axis. The laser pulse is focused in these curved density troughs as shown in Fig. 5(d). As a result, the peak intensity after being compressed is higher than that found in 1D simulations, comparing Fig. 4(c) and Fig. 5(c).

B. Pulse compression in inhomogeneous plasmas

As shown in the last subsection, when the signal pulse is located inside a density trough of the plasma wave, it propagates at a group velocity larger than the phase velocity of the plasma wave. As a result, it tends to propagate out of the bottom of the density trough to a region with increasing density profile. However, the signal pulse can still be trapped inside the density trough if the amplitude of the plasma wave is sufficiently high. One can see this in the propagation frame of the plasma wave. Assuming the frequency of the pump pulse is ω_{pump} in the laboratory frame and the phase velocity of the plasma wave $\beta \approx (1-\omega_p^2/\omega_{pump}^2)^{1/2}$, then the Doppler-

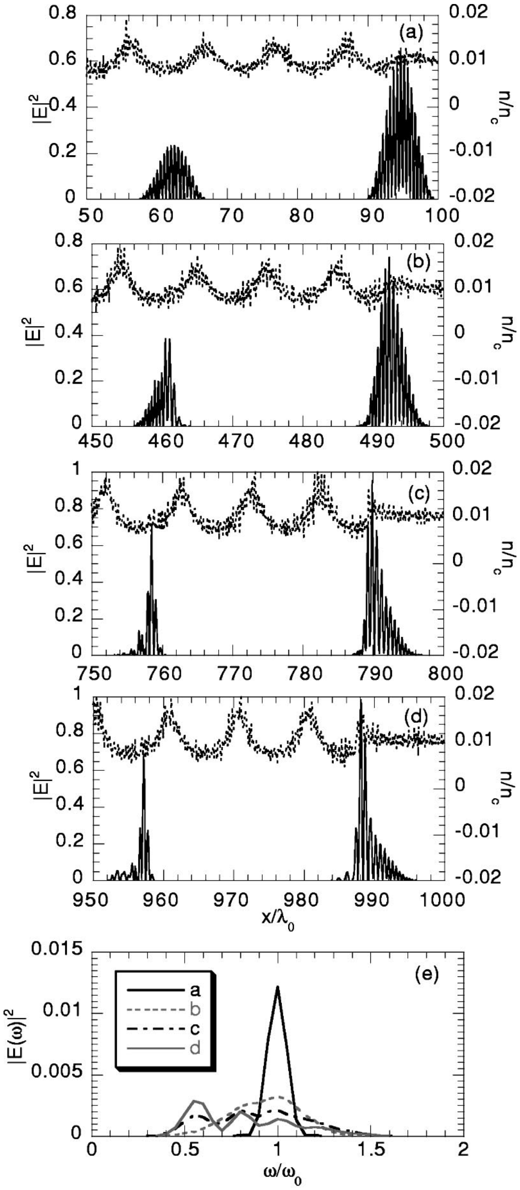

FIG. 6. Pulse compression in inhomogeneous plasma with density changing linearly from $n_0/n_c = 0.01$ to 0.0106 in 1150 λ_0 at $t = 100\tau_0$ (a), $500\tau_0$ (b), $800\tau_0$ (c), and $1000\tau_0$ (d). Initially $a_{01} = 0.8$ and $L_1 = 10\lambda_0$ for the pump pulse and $a_{02} = 0.5$ and $L_2 = 10\lambda_0$ for the signal pulse. Both the signal and pump pulses are in sine profiles with an initial interval of $32.5\tau_0$. The frequency spectra of the signal pulse at these snapshot moments are shown in (e). The transverse field (E) is normalized as in Fig. 2.

shifted frequency of the signal pulse in the propagation frame is $\omega'_{signal} \approx 0.5(\omega_{signal}/\omega_{pump} + \omega_{pump}/\omega_{signal})\omega_p$, where ω_{signal} is the signal frequency in the laboratory frame. The signal pulse can be trapped inside the trough if the peak density (normalized by the unperturbed density) in the plasma wave satisfies $n_{max} > (\omega'_{signal}/\omega_p)^2$. Even though the signal laser pulse can be easily trapped inside the density trough for a signal pulse even with frequency several times the pump pulse, the problem is that if the signal pulse moves out of the bottom of the density trough in a short time, the frequency spectrum of the pulse does not get broadened much before it moves to the region with an increasing density profile, where the phase modulation is effective in producing the frequency downshift, not in broadening the frequency spectrum of the pulse. As a result, the pulse is not compressed efficiently. To make the pulse locate near the bottom of the density trough for as long time as possible for better compression, one may slightly increase the plasma density along the pulse propagation direction or use a signal pulse with a frequency slightly lower than the pump. Here we only consider the first scheme. Figure 6 shows such an example, where we increase the plasma density linearly by only 6% over a distance of $1150\lambda_0$. In comparison with Fig. 3, we find that the pulse can somehow be better compressed in inhomogeneous plasmas.

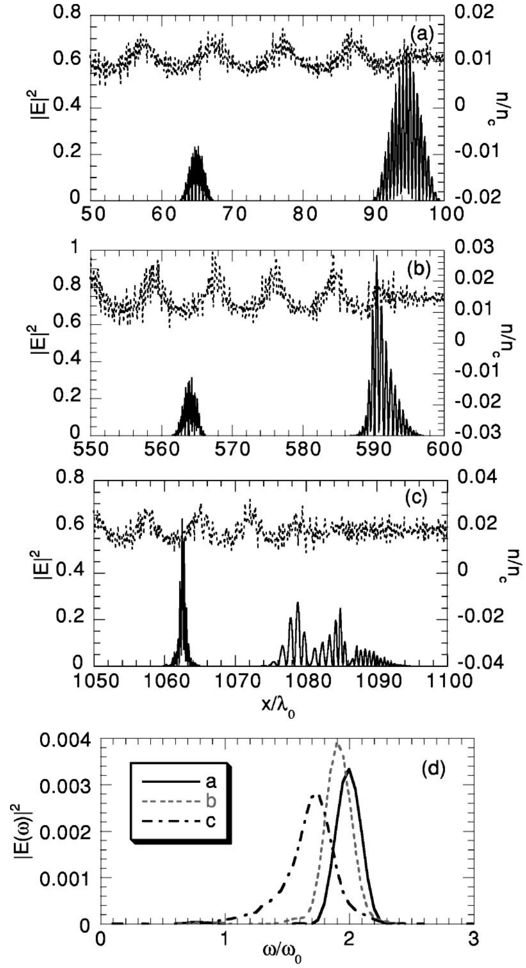

FIG. 7. Pulse compression in inhomogeneous plasma with density changing linearly from $n_0/n_c = 0.01$ to 0.022 in 1550 λ_{pump} at $t = 100\tau_{pump}$ (a), $600\tau_{pump}$ (b), and $1100\tau_{pump}$ (c), where τ_{pump} and λ_{pump} are the laser cycle and wavelength of the pump in vacuum. The frequency of the signal pulse is double that of the pump. Initially $a_{01} = 0.8$ and $L_1 = 10\lambda_{pump}$ for the pump pulse and $a_{02} = 0.5$ and $L_2 = 5\lambda_{pump}$ for the signal pulse. Both the signal and pump pulses are in sine profiles with an initial interval of $32.5\tau_{pump}$. The frequency spectra of the signal pulse at these snapshot moments are shown in (d). The transverse field (E) is normalized by $m\omega_0 c/e$ with ω_0 the initial central frequency of the pump pulse.

In the next example, we consider compressing signal pulses with higher frequencies than that of the pump pulse. Assuming the density profile is $n = n_0[1 + f(x)]$ with $f(x) \ll 1$. The interval of the two pulses can be written as $\Delta x = \Delta x_0 - (\omega_p^2/2\omega_{pump}^2)(1 - \omega_{pump}^2/\omega_{signal}^2)\int_0^x [(1 + f(x')]dx'$, where Δx_0 is the initial interval between the pulses before moving into the plasma, $\omega_p=(4\pi n_0 e^2/m)^{1/2}$, and we assume $\omega_{pump}<\omega_{signal}$. The plasma wavelength changes with the spatial coordinate as $\lambda_p\approx\lambda_{p0}[1+f(x)]^{-1/2}$ with $\lambda_{p0}=\lambda_{pump}(\omega_{pump}/\omega_p)$, where λ_{pump} is the wavelength of the pump pulse. If the initial interval $\Delta x_0=N\lambda_{p0}$ with N some integer, to make the signal pulse remain located near the bottom of the same density trough of the plasma wave, one needs $\Delta x=N\lambda_p$. As a result, one finds

$$1-(\alpha/\lambda_{pump})\int_0^x[(1+f(x')]dx'=(1+f)^{-1/2}, \quad (14)$$

with $\alpha=(\omega_p^3/2N\omega_{pump}^3)(1-\omega_{pump}^2/\omega_{signal}^2)$. From Eq. (14) one finds $1+f(x)=(1-3\alpha x/\lambda_{pump})^{-2/3}\approx 1+2\alpha x/\lambda_{pump}$. If taking $\omega_p^2/\omega_{pump}^2=0.01$, $N=3$, and $\omega_{pump}/\omega_{signal}=1/2$, one gets $\alpha=1.25\times10^{-4}$. Usually this value is found to be less estimated since we have not yet taken into account that the density at the trough is lower than the unperturbed one. In Fig. 7, we show an example when the frequency of the signal pulse is twice that of the pump pulse. In the simulation, the plasma density is increased from $0.01n_c$ up to $0.022n_c$ over $1550\lambda_{pump}$, corresponding to $\alpha=3.9\times10^{-4}$. The signal pulse initially with ten cycles is compressed to less than five cycles and its intensity is doubled after the pulse propagating over $1060\lambda_{pump}$. Similar to the case in homogeneous plasmas, the frequency of compressed signal pulse is downshifted. Finally, the pump pulse decays to low intensity and its pulse width is increased. As a result, the generated wake field is at low amplitude and cannot compress the pulse further more.

IV. CONCLUSIONS AND DISCUSSION

We have studied analytically and numerically the scheme of time compressing a short laser pulse with a high-amplitude plasma wave. In this scheme, we use a pump pulse to generate a background plasma wave and let a signal pulse follow behind. One tunes the time delay between the pulses so that the signal pulse is located inside a density trough of the wake field of the pump. For a given pulse width, it is found analytically that there exists a threshold amplitude of the plasma wave at which its compression effect balances the dispersive spreading of the pulse. Plasma waves with higher amplitudes than the threshold can compress the signal pulse significantly. Both 1D and 2D PIC simulations show that pulse compression by several times down to only about one cycle is possible with a high-amplitude plasma wave. The central frequency of the compressed pulse is usually downshifted owing to the difference between the group velocity of the signal and pump pulses when the signal pulse is located initially at a density trough. Numerical simulations also suggest that inhomogenous plasma densities may allow for better compression and for compression of short pulses with higher frequencies than the pump.

For potential experimental tests of the present scheme in the future, if one takes a titanium:sapphire lasers emitting at 800 nm, a signal pulse initially with ten laser cycles corresponds to a duration of 26.7 fs. Since the duration must be less than a plasma oscillation period, one needs plasma with a density $\sim n_c/10^2=1.12\times10^{19}(\lambda_0/\mu\text{m})^{-2}$ $\text{cm}^{-3}=1.75\times10^{19}$ cm^{-3}. The plasma should be basically homogeneous, extending over about 0.8 mm according to the simulations. If the plasma wave is generated as the wake field of a pump pulse, its peak intensity would be around 1.37×10^{18} W/cm^2 (for $a_0=0.8$) and the pulse energy about 50 mJ if it is focused with a diameter about 20μm and with a duration about the same as the signal pulse for efficient wake field excitation. The delay of the signal pulse with respect to the pump pulse is around 87 fs or longer by several plasma oscillation periods. It appears that most of the parameters given above are technically feasible now.

The present scheme requires, for a signal pulse with initial duration less than $2\pi/\omega_p$, the plasma oscillation period. This appears to be a great limitation for experiments. For a signal pulse with a duration several times the plasma oscillation period, it cannot be compressed as a single pulse, but splits into a pulse train. Simulations show that the relatively long pulse is first modulated by the background plasma wave. This modulated pulse can excite a huge wake field through a mechanism called the self-modulated laser wake field [23] or wake field amplification by a pulse train [24], even though it is at modest intensities. After a certain time comparable for compressing a short pulse, the long pulse is fully split into a pulse train, where each subpulse has a duration of several laser cycles. Therefore, if the signal pulse is longer than $2\pi/\omega_p$, finally one has to separate the produced pulse train in order to get a single ultrashort pulse.

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